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## Radiative rare $B$ decays revisited

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### Abstract

We reexamine contributions of higher  $K$ -resonances to the radiative rare decays  $b \rightarrow s\gamma$  in the limit where both  $b$ - and  $s$ -quark are considered heavy. Using the non-relativistic quark model, and the form factor definitions consistent with the HQET covariant trace formalism, we find significant disagreement with previous work which also used heavy quark symmetry, and excellent agreement with experimental results. In particular, the two largest fractions of the inclusive  $b \rightarrow s\gamma$  branching ratio are found to be  $(16.8 \pm 6.4)\%$  for  $B \rightarrow K^*(892)\gamma$  and  $(6.2 \pm 2.9)\%$  for  $B \rightarrow K_2^*(1430)\gamma$  decays. We also compare the contribution from the radiative decays into the eight  $K$ -meson states to the inclusive experimental  $b \rightarrow s\gamma$  mass distribution.

# 1 Introduction

Flavor changing neutral current transitions involving the  $B$ -meson provide a unique opportunity to study the electroweak theory in higher orders. Although transitions like  $b \rightarrow s\gamma$ ,  $b \rightarrow se^+e^-$ , and  $b \rightarrow sg$  vanish at the tree level, they can be described by one loop (“penguin”) diagrams, in which a  $W^-$  is emitted and reabsorbed [1]. These processes occur at a rate small enough to be sensitive to physics beyond the Standard Model [2]. Similar flavor violating processes in the  $K$ -meson system have the disadvantage that non-perturbative long distance effects are quite large, and it is difficult to extract the quark level physics from well known processes like  $K^+ \rightarrow \pi^+e^+e^-$ .

Among all rare  $B$  decays, radiative processes  $B \rightarrow X_s\gamma$  (especially decay  $B \rightarrow K^*(892)\gamma$ ) have received an increasing attention, because of the experimental measurement of the  $B \rightarrow K^*(892)\gamma$  exclusive branching ratio [3],

$$BR(B \rightarrow K^*(892)\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5} . \quad (1)$$

which has been recently updated [4] to

$$BR(B \rightarrow K^*(892)\gamma) = (4.3^{+1.1}_{-1.0} \pm 0.6) \times 10^{-5} , \quad (2)$$

and also of the inclusive rate [5],

$$BR(B \rightarrow X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} . \quad (3)$$

Several methods have been employed to predict exclusive  $B \rightarrow K^*(892)\gamma$  decay rate: HQET [6, 7], QCD sum rules [8]-[13], quark models [14]-[24], bound state resonances [25], and Lattice QCD [26]-[29]. The theoretical uncertainty, which was originally of two orders of magnitude, has been greatly reduced in the more recent studies. However, there is still a large spread between different results.

In this paper we follow the approach of [6, 7], in which both  $b$ - and  $s$ -quark are considered heavy. In the heavy quark limit the long distance effects are contained

within unknown form factors, whose precise definition consistent with the covariant trace formalism [30]-[32] has been clarified only recently [33]. This is precisely the reason why our results substantially differ from [7], even though we use the same non-relativistic quark model for the wave functions of the light degrees of freedom (LDF). Our results show that the ratio of the exclusive  $B \rightarrow K^{**}\gamma$  to the inclusive decay rate  $B \rightarrow X_s\gamma$  was underestimated for the channel  $B \rightarrow K^*(892)\gamma$  ( $(16.8 \pm 6.4)\%$  as opposed to  $(3.5 - 12.2)\%$  from [7]), and significantly overestimated for the decay  $B \rightarrow K_2^*(1430)\gamma$  ( $(6.2 \pm 2.9)\%$  as opposed to  $(17.3 - 37.1)\%$  from [7]). We emphasize that our prediction for the decay  $B \rightarrow K^*(892)\gamma$  is in agreement with experimental result of  $(19 \pm 5)\%$ . Although other exclusive decays have not yet been identified, we have compared with experiment the contribution from the eight  $B \rightarrow K^{**}\gamma$  decays to the inclusive  $B \rightarrow X_s\gamma$  mass distribution.

The paper is organized as follows: in Section 2 we restate the theoretical framework for the  $B \rightarrow K^{**}\gamma$  decays. Section 3 contains a discussion of the form factor calculation. The expressions for the form factors given in [33] are evaluated in terms of the wave functions and energies of the light degrees of freedom in the meson rest frame. We discuss here the model used in establishing the LDF wave functions and energies. An extensive literature exists in this subject, so we have attempted to set our results in context with previous calculations in Section 4. Our conclusions are summarized in Section 5.

## 2 Theory of $B \rightarrow K^{**}\gamma$ decays

The effective Hamiltonian for the decays  $B \rightarrow X_s\gamma$  can be found in many places, e.g. [34]-[36]. It is derived by integrating out the top quark and  $W$ -boson at the same scale  $\mu \approx M_W$ . An appropriate operator basis for the effective Hamiltonian consists of four-quark operators and the magnetic moment type operators of dimension six ( $\mathcal{O}_1 - \mathcal{O}_8$ ). Higher dimensional operators are suppressed by powers of the masses of

the heavy particles. For the  $B \rightarrow K^{**}\gamma$  decays only the operator  $\mathcal{O}_7$  contributes, so that

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7(m_b) \mathcal{O}_7(m_b) . \quad (4)$$

Here,  $\mathcal{O}_7$  is given by

$$\mathcal{O}_7 = \frac{e}{32\pi^2} F_{\mu\nu} [m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b + m_s \bar{s} \sigma^{\mu\nu} (1 - \gamma_5) b] , \quad (5)$$

with  $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ . The explicit expression for the Wilson coefficient  $C_7(m_b)$  as a function of  $\frac{m_t^2}{M_W^2}$  can be found in [36, 37]. The value of  $C_7$  can be calculated perturbatively at the mass scale  $\mu = M_W$ . The evolution from  $M_W$  down to a mass scale  $\mu = m_b$  introduces large QCD corrections. This procedure also introduces large theoretical uncertainties, primarily due to the choice of the renormalization scale  $\mu$  (taken above as  $m_b$ ), which can be as large as 25% [36].

As proposed in [6, 7], we evaluate the hadronic matrix element of  $\mathcal{O}_7$  between a  $B$ -meson in the initial state, and a generic  $K^{**}$ -meson in the final state, in the heavy quark limit for the  $b$ - and  $s$ -quarks. Matrix elements of bilinear currents of two heavy quarks ( $J(q) = \bar{Q}\Gamma Q$ ) are most conveniently evaluated within the framework of the trace formalism, which was formulated in [30, 31] and generalized to excited states in [32]. Denoting  $\omega = v \cdot v'$ , where  $v$  and  $v'$  are the four-velocities of the two mesons, we have

$$\langle \Psi'(v') | J(q) | \Psi(v) \rangle = \text{Tr}[\bar{M}'(v') \Gamma M(v)] \mathcal{M}(\omega) , \quad (6)$$

where  $M'$  and  $M$  denote matrices describing states  $\Psi'(v')$  and  $\Psi(v)$ ,  $\bar{M} = \gamma^0 M^\dagger \gamma^0$ , and  $\mathcal{M}(\omega)$  represents the LDF. For all transitions considered in this paper matrices  $M$  and  $M'$ , as well as definitions for  $\mathcal{M}(\omega)$ , can be found in [7, 33]. Using (6), we can write

$$\langle K^{**}\gamma | \mathcal{O}_7(m_b) | B \rangle = \frac{e}{16\pi^2} \eta_\mu q_\nu \text{Tr}[\bar{M}'(v') \Omega^{\mu\nu} M(v)] \mathcal{M}(\omega) , \quad (7)$$

where the factor  $q_\nu = m_B v_\nu - m_{K^{**}} v'_\nu$  came from the derivative in the field strength  $F_{\mu\nu}$  of (5),  $\eta_\mu$  is the photon polarization vector, and

$$\Omega^{\mu\nu} = m_B \sigma^{\mu\nu} (1 + \gamma_5) + m_{K^{**}} \sigma^{\mu\nu} (1 - \gamma_5) . \quad (8)$$

Expression (7) can be further simplified using  $\not{v} M(v) = M(v)$ .

Now, using the mass shell condition of the photon ( $q^2 = 0$ ), and polarization sums for spin-1 and spin-2 particles, we obtain the following decay rates [7]:

$$\Gamma(B \rightarrow K^*(892)\gamma) = \Omega |\xi_C(\omega)|^2 \frac{1}{y} [(1-y)^3(1+y)^5(1+y^2)] , \quad (9)$$

$$\Gamma(B \rightarrow K_1(1270)\gamma) = \Omega |\xi_E(\omega)|^2 \frac{1}{y} [(1-y)^5(1+y)^3(1+y^2)] , \quad (10)$$

$$\Gamma(B \rightarrow K_1(1400)\gamma) = \Omega |\xi_F(\omega)|^2 \frac{1}{24y^3} [(1-y)^5(1+y)^7(1+y^2)] , \quad (11)$$

$$\Gamma(B \rightarrow K_2^*(1430)\gamma) = \Omega |\xi_F(\omega)|^2 \frac{1}{8y^3} [(1-y)^5(1+y)^7(1+y^2)] , \quad (12)$$

$$\Gamma(B \rightarrow K^*(1680)\gamma) = \Omega |\xi_G(\omega)|^2 \frac{1}{24y^3} [(1-y)^7(1+y)^5(1+y^2)] , \quad (13)$$

$$\Gamma(B \rightarrow K_2(1580)\gamma) = \Omega |\xi_G(\omega)|^2 \frac{1}{8y^3} [(1-y)^7(1+y)^5(1+y^2)] , \quad (14)$$

$$\Gamma(B \rightarrow K^*(1410)\gamma) = \Omega |\xi_{C_2}(\omega)|^2 \frac{1}{y} [(1-y)^3(1+y)^5(1+y^2)] , \quad (15)$$

$$\Gamma(B \rightarrow K_1(1650)\gamma) = \Omega |\xi_{E_2}(\omega)|^2 \frac{1}{y} [(1-y)^5(1+y)^3(1+y^2)] , \quad (16)$$

where we used abbreviations

$$y = \frac{m_{K^{**}}}{m_B} , \quad (17)$$

$$\Omega = \frac{\alpha}{128\pi^4} G_F^2 m_b^5 |V_{tb}|^2 |V_{ts}|^2 |C_7(m_b)|^2 , \quad (18)$$

and the argument of the Isgur-Wise (IW) functions is fixed by the mass shell condition of the photon ( $q^2 = 0$ ),

$$\omega = \frac{1+y^2}{2y} . \quad (19)$$

Note that in the expressions for the decay rates (9)-(16) given in [7], a factor of  $(1 - y^2)$  was omitted. Also, as observed in [7], since decays into the states belonging to the same spin symmetry doublet are described by the same Isgur-Wise function, and since in the heavy-quark limit the two members of a spin doublet are degenerate in mass, from (9)-(16) one has

$$\Gamma(B \rightarrow K_2^*(1430)\gamma) \approx 3\Gamma(B \rightarrow K_1(1400)\gamma) , \quad (20)$$

$$\Gamma(B \rightarrow K_2(1580)\gamma) \approx 3\Gamma(B \rightarrow K_1(1680)\gamma) . \quad (21)$$

As indicated, these relations are only approximate due to a large breaking of the spin symmetry for the  $s$ -quark.

### 3 Model for the Isgur-Wise functions

As already mentioned, even though we use the same non-relativistic quark model, our calculation differs significantly from [7] in evaluation of the IW form factors needed for the decay rates. Assuming that we can describe heavy-light mesons using a non-relativistic potential model, the rest frame LDF wave functions (with angular momentum  $j$  and its projection  $\lambda_j$ ), can be written as

$$\phi_{j\lambda_j}^{(\alpha L)}(\mathbf{x}) = \sum_{m_L, m_s} R_{\alpha L}(r) Y_{Lm_L}(\Omega) \chi_{m_s} \langle L, m_L; \frac{1}{2}, m_s | j, \lambda_j; L, \frac{1}{2} \rangle , \quad (22)$$

where  $\chi_{m_s}$  represent the rest frame spinors normalized to one,  $\chi_{m'_s}^\dagger \chi_{m_s} = \delta_{m'_s, m_s}$ , and  $\alpha$  represents all other quantum numbers. According to [33], instead of the simple overlap of the two wave functions, the form factor definitions should include a Lorentz invariant factor in front of the overlap of the two wave functions describing the initial and the final states of the LDF. Also, following the suggestion of [38], overlaps of the two LDF wave functions can be done in the Breit frame ( $\mathbf{v} = -\mathbf{v}'$ ), where the boost factors (connecting the moving to rest LDF states) cancel out. All

this leads to the following expressions [33] valid for the non-relativistic quark model<sup>1</sup> (suppressing quantum numbers  $\alpha'$  and  $\alpha$ , and using the notation of [7]):

$$\xi_C(\omega) = \frac{2}{\omega+1} \langle j_0(ar) \rangle_{00}, \quad 0_{\frac{1}{2}}^- \rightarrow (0_{\frac{1}{2}}^-, 1_{\frac{1}{2}}^-), \quad (23)$$

$$\xi_E(\omega) = \frac{2}{\sqrt{\omega^2-1}} \langle j_1(ar) \rangle_{10}, \quad 0_{\frac{1}{2}}^- \rightarrow (0_{\frac{1}{2}}^+, 1_{\frac{1}{2}}^+), \quad (24)$$

$$\xi_F(\omega) = \sqrt{\frac{3}{\omega^2-1}} \frac{2}{\omega+1} \langle j_1(ar) \rangle_{10}, \quad 0_{\frac{1}{2}}^- \rightarrow (1_{\frac{3}{2}}^+, 2_{\frac{3}{2}}^+), \quad (25)$$

$$\xi_G(\omega) = \frac{2\sqrt{3}}{\omega^2-1} \langle j_2(ar) \rangle_{20}, \quad 0_{\frac{1}{2}}^- \rightarrow (1_{\frac{3}{2}}^-, 2_{\frac{3}{2}}^-), \quad (26)$$

where (denoting the energy of the LDF as  $E_{\bar{q}}$ ),

$$a = (E_{\bar{q}} + E'_{\bar{q}}) \sqrt{\frac{\omega-1}{\omega+1}}, \quad (27)$$

and

$$\langle F(r) \rangle_{L'L}^{\alpha'\alpha} = \int r^2 dr R_{\alpha'L'}^*(r) R_{\alpha L}(r) F(r). \quad (28)$$

Note that (23)-(26) include transitions from the ground state into radially excited states. If the two  $j = \frac{1}{2}$  states are the same,  $E'_{\bar{q}} = E_{\bar{q}}$  and  $\xi_C$  is normalized to one. The above expressions should be compared with the ones used in [7] (putting a tilde over the form factors to avoid confusion),

$$\tilde{\xi}_C(\omega) = \langle j_0(\tilde{a}r) \rangle_{00}, \quad (29)$$

$$\tilde{\xi}_E(\omega) = \sqrt{3} \langle j_1(\tilde{a}r) \rangle_{10}, \quad (30)$$

$$\tilde{\xi}_F(\omega) = \sqrt{3} \langle j_1(\tilde{a}r) \rangle_{10}, \quad (31)$$

$$\tilde{\xi}_G(\omega) = \sqrt{5} \langle j_2(\tilde{a}r) \rangle_{20}, \quad (32)$$

with the definition

$$\tilde{a} = E'_{\bar{q}} \sqrt{\omega^2-1}. \quad (33)$$

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<sup>1</sup>As pointed out in [33], models based on the Dirac equation with a central potential lead to the same expressions for the IW functions.

For the numerical estimates we employ the model used in [39] (usually referred to as the ISGW model), the Schrödinger equation with

$$V(r) = -\frac{4\alpha_s}{3r} + c + br . \quad (34)$$

With sensible choice of parameters, this simple model gives quite reasonable spin-averaged spectra of  $b\bar{d}$  and  $s\bar{d}$  mesons up to  $L = 2$ . However, instead of just using a single harmonic oscillator wave function (as was done in [7]), for the radial wave function of the LDF, we numerically solve the Schrödinger equation. To determine the parameters of the model, we fix  $b = 0.18 \text{ GeV}^2$  (which was also used in [39]), and vary  $\alpha_s$  and  $c$  for a given value of  $m_{u,d}$  (in the range  $0.30 - 0.35 \text{ GeV}$ ), and  $m_s$  (in the range  $0.5 - 0.6 \text{ GeV}$ ), until a good description of the spin averaged spectra of  $K$ -meson states is obtained. Following this procedure, our  $\alpha_s$  ranges from 0.37 to 0.48, while  $c$  takes values from  $-0.83 \text{ GeV}$  to  $-0.90 \text{ GeV}$ . These parameters are in good agreement with the original ISGW values [39] ( $\alpha_s = 0.50$  and  $c = -0.84 \text{ GeV}$  for  $m_{u,d} = 0.33 \text{ GeV}$  and  $m_s = 0.55$ ). We emphasize that the original ISGW parameters give results that are well inside the ranges for all decays quoted in this paper. By varying the  $c$ - and  $b$ -quark masses we could also obtain good spin averaged description of the  $B$  and  $D$  mesons. However, to be consistent with heavy quark symmetry, the wave function for the  $B$  meson was chosen to be the same as the one obtained for the spin averaged (ground state for  $L = 0$ )  $K$  and  $K^*(892)$  mesons.

To completely define our procedure, we have to specify how the LDF energy  $E_{\bar{q}}$  was determined. In [7] for a given  $K^{**}$ -meson the LDF energy was defined as

$$E_{\bar{q}} = \frac{m_{K^{**}} * m_{u,d}}{m_s + m_{u,d}} . \quad (35)$$

This definition was proposed to account for the fact that  $s$  mesons aren't particularly heavy. On the other hand, a definition that is consistent with heavy quark symmetry is

$$E_{\bar{q}} = m_{K^{**}} - m_s . \quad (36)$$



It should be noted that these two expressions are not equivalent in the heavy quark limit. In order to explore the sensitivity of our results on the choice of  $E_{\bar{q}}$ , we have repeated all calculations employing both of these two definitions, and in the final results we have quoted the broadest possible range obtained for the form factors (and for all other results). Finally,  $E_{\bar{q}}$  for the  $B$  meson has been taken to be the same as  $E_{\bar{q}}$  for the  $K^*(892)$  meson, consistent with heavy quark symmetry. It turns out that this is actually a very reasonable assumption. The range of  $E_{\bar{q}}$  that was used here for  $B$  and  $K^*(892)$  meson was from  $0.296 \text{ GeV}$  to  $0.396 \text{ GeV}$ . On the other hand, from the *CLEO* data on the semileptonic  $B$  decays [40], and the LQCD heavy-light wave function [41], it was estimated [42] that in  $B$  systems  $E_{\bar{q}}$  ranges from  $0.266 \text{ GeV}$  to  $0.346 \text{ GeV}$ .

We believe that the procedure outlined above enables us to estimate a reasonable range for the unknown IW form factors in a physically more acceptable way than it was done in [7], by simply varying the scale parameter of the single harmonic oscillator wave function.

## 4 Our results and comparison with previous investigations

In Table 1 we present our results for the range of (absolute) values of the form factors at the indicated value of  $\omega$ , for the ratio  $R = \frac{\Gamma(B \rightarrow K^{**}\gamma)}{\Gamma(B \rightarrow X_s \gamma)}$ , and for the branching ratio  $BR(B \rightarrow K^{**}\gamma)$ , for the various  $K^{**}$ -mesons. The inclusive branching ratio  $B \rightarrow X_s \gamma$  is usually taken to be QCD improved quark decay rate for  $b \rightarrow s \gamma$ , which can be written as [34, 36, 37]

$$\Gamma(B \rightarrow X_s \gamma) = 4\Omega(1 - \frac{m_s^2}{m_b^2})^3(1 + \frac{m_s^2}{m_b^2}) . \quad (37)$$

The leading log prediction for  $BR(b \rightarrow s \gamma)$  is  $(2.8 \pm 0.8) \times 10^{-4}$  [36, 37], where the uncertainty is due to the choice of the QCD scale. The next-to-leading order terms

that have been calculated tend to reduce the prediction to about  $1.9 \times 10^{-4}$  [27]. Both of these predictions are in excellent agreement with the recent experimental result of  $BR(b \rightarrow s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$  [5]. For the numerical values of the  $B \rightarrow K^{**}\gamma$  branching ratios given in Table 1 we used the leading log result of  $BR(b \rightarrow s\gamma) = 2.8 \times 10^{-4}$ .

In order to make comparison of our results with previous calculations easier, we have tabulated our results together with results of [7] and [16] in Table 2. As far as we know, these two papers are the only ones that have dealt with radiative rare  $B$  decays into higher  $K$ -resonances. There has been much more work done on the decay  $B \rightarrow K^*(892)\gamma$ , and we have tabulated some of these results in Table 3. As one can see from Table 3, the predictions for this particular ratio ranges from a 0.7% [22] to 97.0% [14]. The data suggest a value of  $(19 \pm 5)\%$ . Note that our result of  $(16.8 \pm 6.4)\%$  is consistent with the data, unlike the values quoted in [7] and [16]. As far as decays into higher  $K$  resonances are concerned, our results are in general in much better agreement with [16] than with [7]. In particular, the authors of [7] emphasized a large branching ratio for the decay  $B \rightarrow K_2^*(1430)\gamma$  ( $(17.3 - 37.1)\%$ ), while our results indicate a 3-6 times smaller value of  $(6.2 \pm 2.9)\%$ , a result which agrees with the one quoted in [16] (6.0%). Also note that our numerical results from Table 2 support relations (20) and (21).

With the exception of the  $K^*(892)\gamma$  channel, no other exclusive radiative processes have been identified so far. The inclusive radiative  $B \rightarrow X_s\gamma$  mass distribution has however been measured by *CLEO* [5], and is shown in Fig. 1. We have normalized experimental data so that the integrated distribution gives unity. The  $K^*(892)$  peak is evident, but the higher mass contribution are not resolved. We have attempted to model this inclusive distribution by considering the contributions from each of the exclusive  $K^{**}\gamma$  channels considered in this paper (and given in Table 2).

In order to compare our result to experiment, we replace a given  $R_{K^{**}}$  by a mass

distribution reflecting the finite total width  $\Gamma_{K^{**}}$  of the  $K^{**}$  resonance [43],

$$\frac{dR(m_{X_s})}{dm_{X_s}} = \sum_{K^{**}} \frac{R_{K^{**}}}{\pi} \frac{\Gamma_{K^{**}}/2}{(m_{X_s} - m_{K^{**}})^2 + (\Gamma_{K^{**}}/2)^2} . \quad (38)$$

The integrated distribution gives

$$\int \frac{dR(m_{X_s})}{dm_{X_s}} dm_{X_s} = \sum_{K^{**}} R_{K^{**}} . \quad (39)$$

In Figure 1 we show the total resonance contribution (solid line) compared to the experimental inclusive  $B \rightarrow X_s \gamma$  mass distribution. The area of the resonance curve is 37.4% of the total inclusive rate (see Tables 1 or 2). We see the general shape is correct, but it is difficult to make more quantitative statements due to the large errors involved.

## 5 Conclusion

In this paper we have reexamined predictions of heavy quark symmetry for the radiative rare decays of  $B$ -mesons into higher  $K$ -resonances. An earlier calculation [7] suggested a substantial fraction ((17.3–37.1)%) of the inclusive  $b \rightarrow s \gamma$  branching ratio going into the  $K_2^*(1430)$  channel, and only (3.5–12.2)% going into the  $K^*(892)$  channel. Even though we used the same non-relativistic quark model, our calculation yields fractions of  $(16.8 \pm 6.4)\%$  and  $(6.2 \pm 2.9)\%$  for  $K^*(892)$  and  $K_2^*(1430)$  channels, respectively. Note that experimental results favor the value of  $(19 \pm 5)\%$  for the  $K^*(892)$  channel. Besides a more careful treatment of the uncertainty in the wave functions of the light degrees of freedom, our calculation differs from [7] in employing form factor definitions that are consistent with the HQET covariant trace formalism [33]. As a consequence of that, our results for all decay channels significantly differ from [7]. The contribution of the eight  $K^{**} \gamma$  channels to the inclusive  $B \rightarrow X_s \gamma$  mass distribution was compared with experiment. We find the general shape of the

mass spectrum to be correct, but due to the large errors involved one cannot reach more quantitative conclusions.

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# TABLES

Table 1: Our results for the range of absolute values of the form factors at indicated value of  $\omega$ , for the ratio  $R = \frac{\Gamma(B \rightarrow K^{**}\gamma)}{\Gamma(B \rightarrow X_s \gamma)}$ , and for the branching ratio  $BR(B \rightarrow K^{**}\gamma)$ , for the various  $K^{**}$ -mesons. For the calculation of branching ratios we used the value  $\Gamma(B \rightarrow X_s \gamma) = 2.8 \times 10^{-4}$  [36].

Meson	State	$J^P$	$j$	$\omega$	$\xi$	$R[\%]$	$BR \times 10^5$
$K$	$C$	$0^-$	$\frac{1}{2}$			forbidden	
$K^*(892)$	$C^*$	$1^-$	$\frac{1}{2}$	3.031	$0.289 \pm 0.057$	$16.8 \pm 6.4$	$4.71 \pm 1.79$
$K^*(1430)$	$E$	$0^+$	$\frac{1}{2}$			forbidden	
$K_1(1270)$	$E^*$	$1^+$	$\frac{1}{2}$	2.194	$0.277 \pm 0.053$	$4.3 \pm 1.6$	$1.20 \pm 0.44$
$K_1(1400)$	$F$	$1^+$	$\frac{3}{2}$	2.016	$0.171 \pm 0.040$	$2.1 \pm 0.9$	$0.58 \pm 0.26$
$K_2^*(1430)$	$F^*$	$2^+$	$\frac{3}{2}$	1.987	$0.175 \pm 0.043$	$6.2 \pm 2.9$	$1.73 \pm 0.80$
$K^*(1680)$	$G$	$1^-$	$\frac{3}{2}$	1.702	$0.241 \pm 0.035$	$0.5 \pm 0.2$	$0.15 \pm 0.04$
$K_2(1580)$	$G^*$	$2^-$	$\frac{3}{2}$	1.820	$0.203 \pm 0.024$	$1.7 \pm 0.4$	$0.46 \pm 0.11$
$K(1460)$	$C_2$	$0^-$	$\frac{1}{2}$			forbidden	
$K^*(1410)$	$C_2^*$	$1^-$	$\frac{1}{2}$	2.003	$0.175 \pm 0.014$	$4.1 \pm 0.6$	$1.14 \pm 0.18$
$K_0^*(1950)$	$E_2$	$0^+$	$\frac{1}{2}$			forbidden	
$K_1(1650)$	$E_2^*$	$1^+$	$\frac{1}{2}$	1.756	$0.229 \pm 0.040$	$1.7 \pm 0.6$	$0.47 \pm 0.16$
total						$37.4 \pm 13.6$	$10.44 \pm 3.78$



Table 2: Comparison of our results for the ratio  $R = \frac{\Gamma(B \rightarrow K^{**}\gamma)}{\Gamma(B \rightarrow X_s \gamma)}$  with the previous work done in [7] and [16]. Note that in the quark model calculations decay into the  $^1P_1$  state is forbidden, because  $\mathcal{O}_7$  is a spin-flip operator, and  $K_1(1270)$  and  $K_1(1400)$  are mixtures of  $^1P_1$  and  $^3P_1$  states. In [16]  $^3P_1$  state had  $R = 6\%$ .

Meson	State	$J^P$	$R[\%]$ (this work)	$R[\%]$ (ref. [7])	$R[\%]$ (ref. [16])
$K$	$C$	$0^-$		forbidden	
$K^*(892)$	$C^*$	$1^-$	$16.8 \pm 6.4$	$3.5 - 12.2$	4.5
$K^*(1430)$	$E$	$0^+$		forbidden	
$K_1(1270)$	$E^*$	$1^+$	$4.3 \pm 1.6$	$4.5 - 10.1$	forbidden/6.0
$K_1(1400)$	$F$	$1^+$	$2.1 \pm 0.9$	$6.0 - 13.0$	forbidden/6.0
$K_2^*(1430)$	$F^*$	$2^+$	$6.2 \pm 2.9$	$17.3 - 37.1$	6.0
$K^*(1680)$	$G$	$1^-$	$0.5 \pm 0.2$	$1.0 - 1.5$	0.9
$K_2(1580)$	$G^*$	$2^-$	$1.7 \pm 0.4$	$4.5 - 6.4$	4.4
$K(1460)$	$C_2$	$0^-$		forbidden	
$K^*(1410)$	$C_2^*$	$1^-$	$4.1 \pm 0.6$	$7.2 - 10.6$	7.3
$K_0^*(1950)$	$E_2$	$0^+$		forbidden	
$K_1(1650)$	$E_2^*$	$1^+$	$1.7 \pm 0.6$	not given	not given
total			$37.4 \pm 13.6$	$44.1 - 90.9$	29.1

Table 3: Comparison of our results for the ratio  $R = \frac{\Gamma(B \rightarrow K^* \gamma)}{\Gamma(B \rightarrow X_s \gamma)}$  with several previous calculations.

Author(s)	Ref.	$R[\%]$
O'Donnell (1986)	[14]	97.0
Deshpande et al. (1988)	[15]	6.0
Dominguez et al. (1988)	[8]	$28.0 \pm 11.0$
Altomari (1988)	[16]	4.5
Deshpande et al. (1989)	[19]	6.0-14.0
Aliev et al. (1990)	[9]	39.0
Ali et al. (1991)	[6]	28.0-40.0
Du et al. (1992)	[20]	69.0
Faustov et al. (1992)	[21]	6.5
El-Hassan et al. (1992)	[22]	0.7-12.0
O'Donnell et al. (1993)	[17]	10.0
Colangelo et al. (1993)	[10]	$17.0 \pm 5.0$
Ali et al. (1993)	[7]	$3.5 \pm 12.2$
Ali et al. (1993)	[12]	$16.0 \pm 5.0$
Ali et al. (1993)	[18]	$13.0 \pm 3.0$
Ball (1994)	[11]	$20.0 \pm 6.0$
Narison (1994)	[13]	$16.0 \pm 4.0$
Holdom et al. (1994)	[23]	$17.0 \pm 4.0$
Atwood et al. (1994)	[25]	1.6-2.5
Bernard et al. (1994)	[26]	$6.0 \pm 1.2 \pm 3.4$
Ciuchini et al. (1994)	[27]	$23.0 \pm 9.0$
Bowler et al. (1994)	[28]	$9.0 \pm 3.0 \pm 1.0$
Burford et al. (1995)	[29]	15.0-35.0
Tang et al. (1995)	[24]	10.0-12.0
this work		$16.8 \pm 6.4$

## FIGURES

Figure 1: The experimental inclusive  $B \rightarrow X_s \gamma$  mass distribution measured at *CLEO* [5]. The data have been normalized to unity. The curve is the sum of the exclusive  $K^{**} \gamma$  channels from Table 1 as calculated by (38).

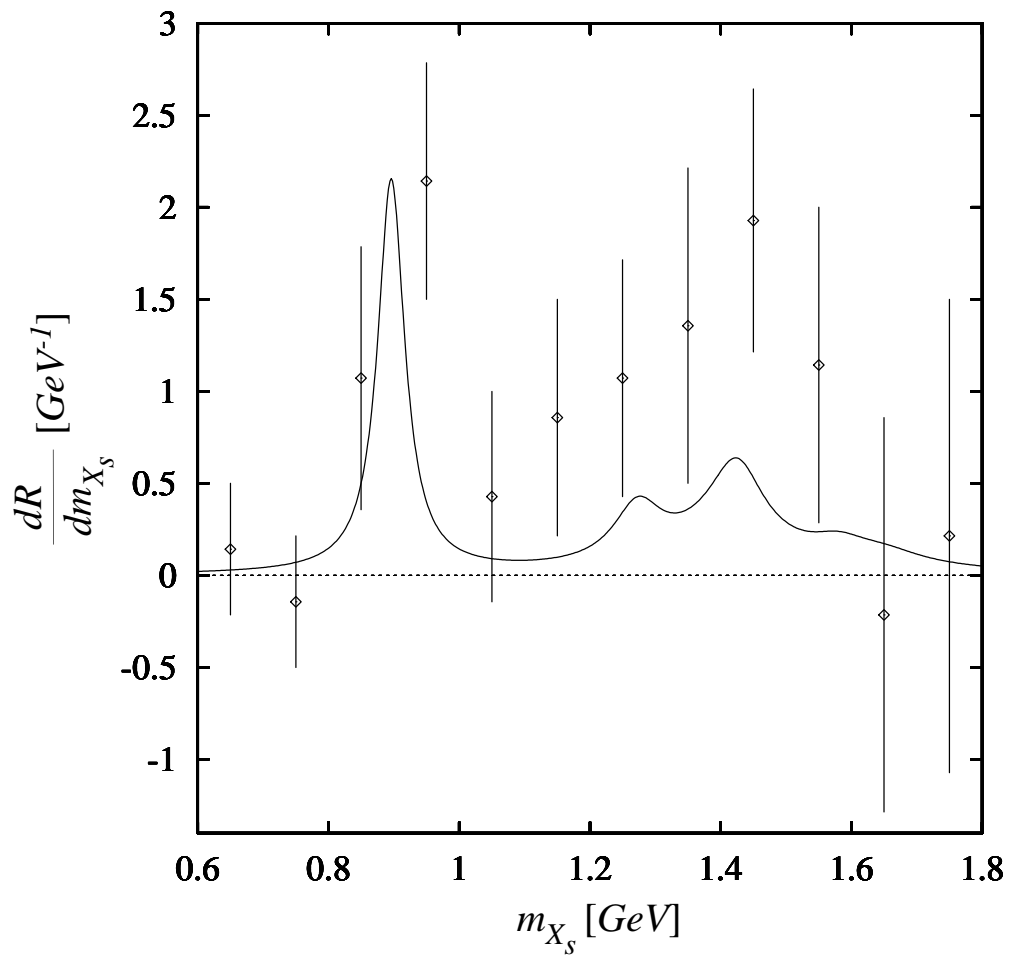


Figure 1